

Isospin fractionation and isoscaling in dynamical simulations of nuclear collisions

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(Received 8 May 2003; published 19 November 2003)

Isospin fractionation and isoscaling are found to hold for isotopes from nuclear collisions simulated within the antisymmetrized molecular dynamics model. The observed linear relation between the isoscaling parameter and the fragment isospin asymmetry $(Z/A)^2$ of the liquid corroborates, in particular, the applicability of statistical considerations to the dynamical fragment emission. Using the derived relation, we are able to extract statistical parameters from the outcome of the dynamic evolution of collisions. The relation allows to access the symmetry energy in the equation of state. This is consistent with the previous findings that the isoscaling parameters can be used to limit the symmetry energy parameter in the nuclear equation of state.

DOI: 10.1103/PhysRevC.68.051601

PACS number(s): 25.70.Pq

In typical intermediate-energy nuclear collisions, numerous fragments of intermediate size are produced, in addition to light particles [1]. The multifragmentation phenomenon is believed to be related to the liquid-gas phase coexistence in low density expanding nuclear matter. In an equilibrated two-component system with more neutrons than protons ($N^{\text{tot}} > Z^{\text{tot}}$), the gas phase becomes more neutron-rich than the liquid phase [2]. This fractionation phenomenon should reflect the features of the symmetry energy in nuclear matter.

Recently, a scaling relation

$$Y_2(N, Z)/Y_1(N, Z) \propto e^{\alpha N + \beta Z} \quad (1)$$

has been observed [3] for the measured fragment yields $Y_i(N, Z)$ from two similar systems $i=1, 2$ with different neutron-to-proton ratios. This phenomenon is called isoscaling. If one assumes thermal and chemical equilibrium, the isoscaling parameters α and β are related to the neutron-proton content of the emitting source. In fact, statistical models have successfully explained the isoscaling data [4]. However, as fragments are formed during a dynamical evolution of the collision system, multifragmentation should be understood in the dynamical models as well. Some dynamical models such as stochastic mean field model predict very large scaling parameters for the dynamically produced fragments [5], which do not agree with the statistical predictions. In this paper, we study this problem by employing the antisymmetrized molecular dynamics (AMD) model. Our results show not only that isoscaling is found in the AMD simulations, but also that the isoscaling and fractionation phenomena follow statistical relations. This opens up direct possibilities to exploit statistical concepts in analyzing dynamically produced fragments. We will show that the scaling parameters from analysis can be directly related to the asymmetry term of the equation of state (EOS) of nuclear matter. To study these issues, we perform AMD simulations for the collisions of Ca isotopes at zero impact parameter using two different symmetry interactions. To emphasize the isospin

effects, we choose the collisions of $^{40}\text{Ca}+^{40}\text{Ca}$, $^{48}\text{Ca}+^{48}\text{Ca}$, and $^{60}\text{Ca}+^{60}\text{Ca}$ with large isospin range.

AMD is a microscopic model for following the time evolution of nuclear collisions [6–8]. The colliding system is represented within the model in terms of a fully antisymmetrized product of Gaussian wave packets. Through the time evolution, the wave packet centroids move according to a deterministic equation of motion. Besides, the followed state of the simulation branches stochastically and successively into a huge number of reaction channels. The branching is caused by the two-nucleon collisions and by the splittings of the wave packets. The interactions are parametrized in the AMD model in terms of an effective force acting between nucleons and in terms of the two-nucleon collision cross sections.

We perform reaction simulations employing two different effective forces in order to study effects of the asymmetry term within the forces. One is the usual Gogny force [9], consistent with the saturation of symmetric nuclear matter at the incompressibility $K=228$ MeV. The force is composed of finite-range two-body terms and of a density-dependent term of the form $t_3\rho^{1/3}(1+P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2)$, where P_σ is the spin exchange operator and t_3 is a coefficient. The second force (called Gogny-AS force) is obtained by modifying the Gogny force with

$$V_{\text{Gogny-AS}} = V_{\text{Gogny}} - (1-x)t_3[\rho(\mathbf{r}_1)^{1/3} - \rho_0^{1/3}]P_\sigma\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (2)$$

where $x=-\frac{1}{2}$ and $\rho_0=0.16$ fm $^{-3}$. The two forces coincide at $\rho=\rho_0$. Furthermore, they produce the same EOS of symmetric nuclear matter at all densities. However, the two forces produce different density dependences of the symmetry energy, as shown in Fig. 1. The choice of $x=-\frac{1}{2}$ has been made to ensure that the part of the symmetry energy from the direct interaction term is proportional to the density [10]. At densities below ρ_0 , the Gogny force has somewhat higher symmetry energy than the Gogny-AS force. At densities above ρ_0 , the Gogny-AS symmetry en-

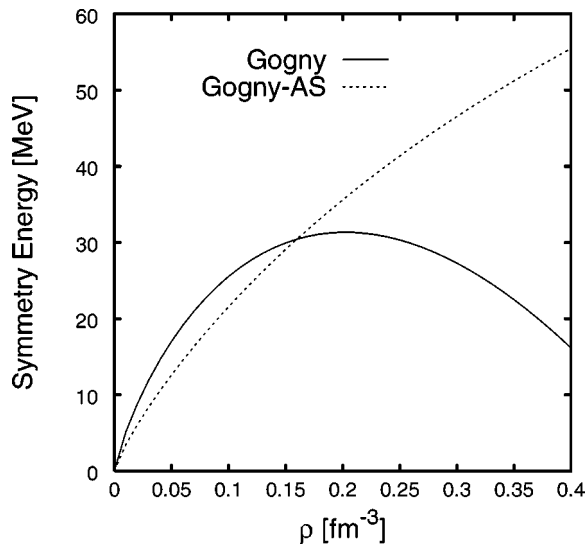


FIG. 1. Density dependence of the symmetry energy of nuclear matter for the Gogny force (solid line) and for the Gogny-AS force (dashed line).

ergy continues to rise while the Gogny symmetry energy begins to fall, so that significant differences develop. However, the difference in the density dependence of symmetry energy for these forces is not as extreme as for those used in other works such as Ref. [5].

The version of AMD of Ref. [8] was utilized. It has been demonstrated that an equivalent version of AMD, for the present purposes, reproduces the experimental data of various fragment observables in $^{40}\text{Ca}+^{40}\text{Ca}$ at the same energy of 35 MeV/nucleon with the Gogny force [7,11]. In the present study, each collision event was started by boosting two nuclei with centers separated by 9 fm. The dynamical simulation was continued until $t=300$ fm/c. About 1000 events were generated for each combination of nuclei.

In central collisions, as shown in a previous paper [7,11], two nuclei basically penetrate each other and many fragments are formed not only from the projectilelike and targetlike parts but also from within the neck region between the two residues. The nuclear matter appears to be strongly expanding, one dimensionally, in the beam direction.

To study the isospin fractionation effect where the neutron and proton densities are distributed inhomogeneously between liquid and gas phase, we define the liquid part as the part of the system composed of the fragments with $A > 4$, with any two wave packets of spatial separation less than 3 fm treated as belonging to the same fragment. Figure 2 shows the time evolution of the isospin asymmetry $(Z/A)^2$ of the liquid part for the three reaction systems. At the initial value ($t \sim 0$), $(Z/A)_{\text{liq}}^2$ is $(Z/A)^2$ of the initial nuclei. For the neutron-rich systems, $(Z/A)_{\text{liq}}^2$ increases rapidly before $t \sim 100$ fm/c, and then it continues to increase only gradually. This effect can be regarded as the isospin fractionation because the liquid part is getting less neutron rich and the gas part is getting more neutron rich. Such fractionation effect is much more dramatic in the gas part. Figure 3 shows the neutron and proton emission rates for the three systems. For the very neutron-rich system ($^{60}\text{Ca}+^{60}\text{Ca}$) many more neu-

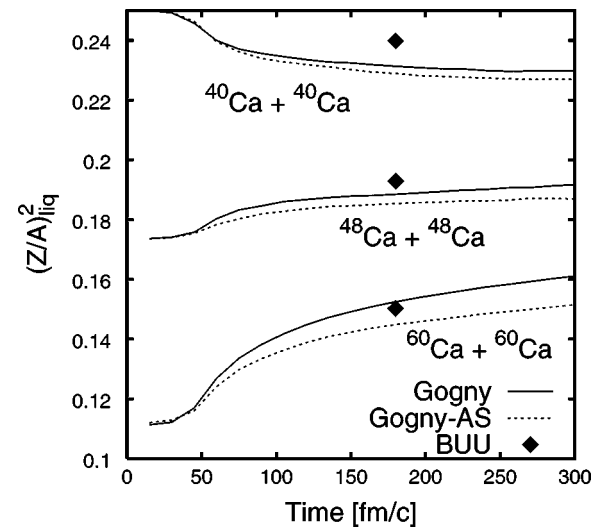


FIG. 2. $(Z/A)^2$ of the liquid part of the system as a function of time for the three reaction systems. The AMD results are represented by the solid and dashed lines, respectively, for the Gogny and Gogny-AS forces. Late-time BUU results are represented by filled diamonds.

trons are emitted compared to nearly zero proton emission suggesting the existence of a very neutron-rich gas. This is contrasted with the $^{40}\text{Ca}+^{40}\text{Ca}$ system where more protons are emitted than neutrons.

Given that neutron emission costs less energy in a more neutron-rich system and proton emission costs more, it is not quite surprising that the isospin fractionation is observed in the dynamical simulations. Similar fractionation effects have been observed in other dynamical model simulations [10,13]. To test the dependence of the results on differences between transport models, we have performed the Boltzmann-Uehling-Uhlenbeck (BUU) calculations [12] with an interaction symmetry energy of $12.125(\rho/\rho_0)$ MeV. The results are shown as the diamond points in Fig. 2. Both the BUU and the AMD models predict similar trends in the asymmetry of the liquid.

Figure 2 suggests that the isospin fractionation has a clear dependence on the asymmetry term of the effective force. The Gogny force (solid lines) always yields a system with a larger $(Z/A)_{\text{liq}}^2$ than the Gogny-AS force (dashed lines). In Ref. [3], the isospin fractionation effect has been studied using the isoscaling observable involving isotope yield ratios, under the assumption that the fragments are formed in equilibrium described within the grand canonical approach. Since fragments are produced within a rapidly evolving system in the AMD simulation, it is not evident *a priori* whether the isoscaling [Eq. (1)] needs to be expected there in the fragment yield ratio. Nevertheless, when we plot the fragment yield ratio, $Y_2(N, Z)/Y_1(N, Z)$, from two reaction systems, we observe a clear isoscaling relation, as shown in Figs. 4 and 5 obtained for the fragments present at $t = 300$ fm/c. The extracted scaling parameters α and β are provided in the figure captions and additionally indicated in the individual panels. The isoscaling parameters, α and β , clearly depend on the asymmetry term of the effective force. Furthermore, their magnitude increases with increased differ-

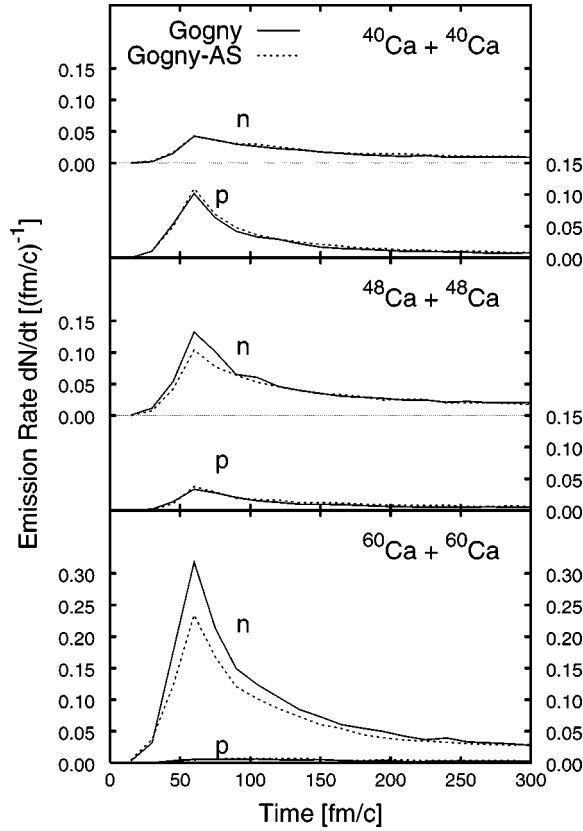


FIG. 3. Neutron and proton emission rates described by the left- and right-hand scales, respectively, for the three reaction systems as a function of time. The results of AMD simulations with the Gogny and Gogny-AS forces are, respectively, represented by the solid and dashed lines.

ences in the asymmetry of the two systems.¹

Since both the isoscaling parameters α and the asymmetry of the “liquid” $(Z/A)_{\text{liq}}^2$ are sensitive to the symmetry term of the effective force, we examine their relationship in Fig. 6 for the three reaction systems. A linear relationship between α and $(Z/A)_{\text{liq}}^2$ is observed and the slope for the relation depends on the symmetry terms used. If the slopes are proportional to the symmetry energy, as suggested by the consideration below, the ratio of $C(\text{Gogny})/C(\text{Gogny-AS}) \approx 1.25$ is consistent with the idea that fragmentation occurs at low density, $\rho < \rho_0$ where $C(\text{Gogny}) > C(\text{Gogny-AS})$. From Fig. 1, this further implies that fragments are formed when $\rho \sim 0.08 \text{ fm}^{-3}$.

The isoscaling evident in Figs. 4 and 5 is a nontrivial result, difficult to explain outside of statistical considerations. The linear relationship in Fig. 6 between α and $(Z/A)_{\text{liq}}^2$ further strengthens the evidence that statistical laws

¹If the values of the isoscaling parameters, obtained in the present work, are scaled down by the ratio of the difference in isospin asymmetry [$\Delta(N/Z)=1$] between $^{40}\text{Ca}+^{40}\text{Ca}$ and $^{60}\text{Ca}+^{60}\text{Ca}$ systems, to the difference in isospin between the experimental systems [3] of $^{112}\text{Sn}+^{112}\text{Sn}$ and $^{124}\text{Sn}+^{124}\text{Sn}$, we obtain the α value of 0.43. This is not far from the experimental value of Ref. [3], 0.36, and a lot lower than the value of 1.07 obtained in the stochastic mean field calculation of Ref. [5].

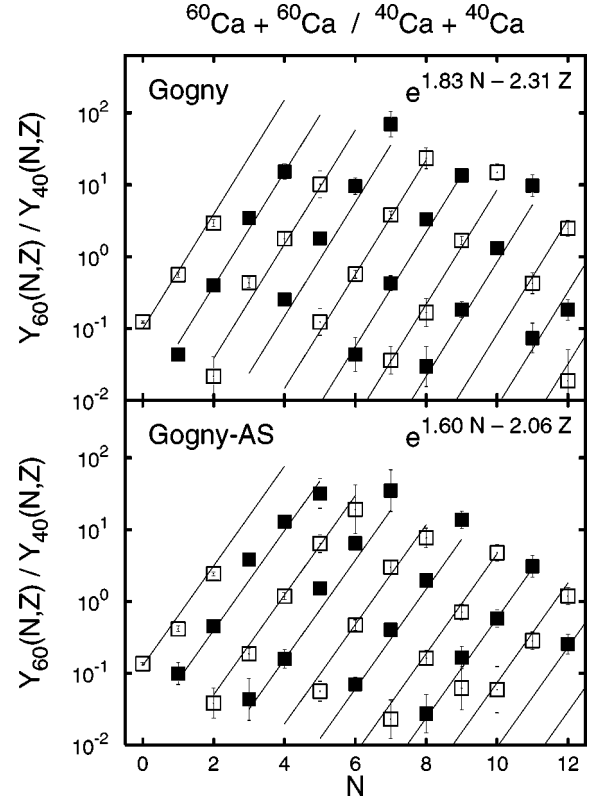


FIG. 4. The fragment yield ratio between the AMD simulations of central $^{60}\text{Ca}+^{60}\text{Ca}$ and $^{40}\text{Ca}+^{40}\text{Ca}$ collisions at 35 MeV/nucleon, at time $t=300 \text{ fm/c}$. The top and bottom panels show, respectively, the results obtained using the Gogny and Gogny-AS forces. The extracted isoscaling parameters are $\alpha=1.83\pm 0.03$ and $\beta=-2.31\pm 0.04$ for the Gogny force, and $\alpha=1.60\pm 0.02$ and $\beta=-2.06\pm 0.03$ for the Gogny-AS force.

may be applicable to the isospin composition of fragments. To explore this issue, we derive below how the isoscaling parameter and the fragment isospin asymmetry should be related, if equilibrium is assumed in an ensemble of fragments similar to those found after the dynamical fragmentation stage in the AMD simulations.

When we consider a system in equilibrium, at the temperature T and pressure P , the number (or yield) of a nucleus composed of N neutrons and Z protons is given by

$$Y_i(N, Z) = Y_{0i} \exp\{-[G_{\text{nuc}}(N, Z) - \mu_{ni}N - \mu_{pi}Z]/T\}, \quad (3)$$

where the index i specifies the reaction system, with the total neutron and proton numbers N_i^{tot} and Z_i^{tot} , and $G_{\text{nuc}}(N, Z)$ stands for the internal Gibbs free energy of the (N, Z) nucleus. The net Gibbs free energy G_{tot} for the system is related to the chemical potentials μ_{ni} and μ_{pi} by $G_{\text{tot}} = \mu_{ni}N_i^{\text{tot}} + \mu_{pi}Z_i^{\text{tot}}$. In Eq. (3) and the following equations, we suppress the (T, P) dependence for different quantities including G_{nuc} , μ_{ni} , and μ_{pi} . It is clear that isoscaling [Eq. (1)] is satisfied for Eq. (3), with $\alpha = (\mu_{n2} - \mu_{n1})/T$ and $\beta = (\mu_{p2} - \mu_{p1})/T$, as long as the two systems have common temperature and pressure.

For each given Z , the dependence of G_{nuc} on N , assuming gradual changes, takes the form

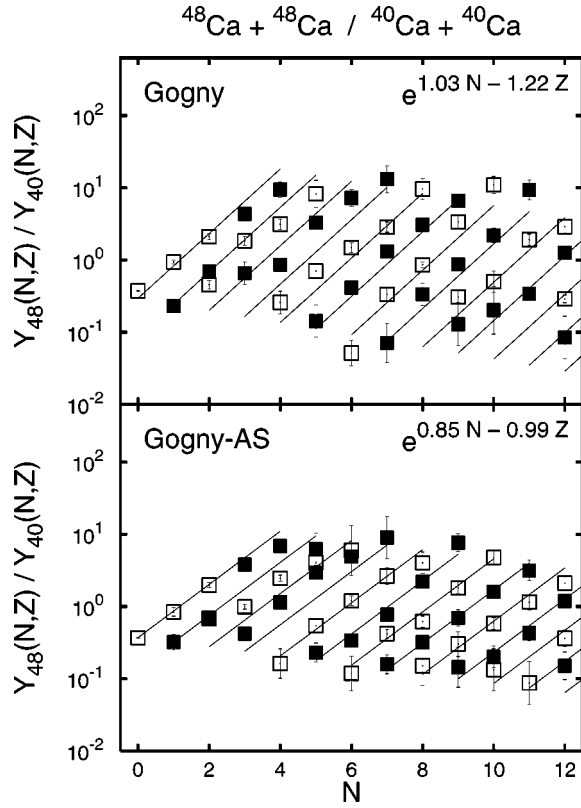


FIG. 5. The same as Fig. 4 but for the $^{48}\text{Ca}+^{48}\text{Ca}$ and $^{40}\text{Ca}+^{40}\text{Ca}$ collisions. The extracted isoscaling parameters are $\alpha = 1.03 \pm 0.02$ and $\beta = -1.22 \pm 0.02$ for the Gogny force, and $\alpha = 0.84 \pm 0.01$ and $\beta = -0.99 \pm 0.02$ for the Gogny-AS force.

$$G_{\text{nuc}}(N, Z) = a(Z)N + b(Z) + C(Z)(N - Z)^2/A. \quad (4)$$

Because the important range of N is limited for a given Z , this expansion is practically sufficient even when G_{nuc} contains surface terms, Coulomb terms, and any other terms which are smooth in A , such as, e.g., the term $\tau T \ln A$ introduced by Fisher [15]. We can regard $C(Z)$ as the symmetry energy coefficient in the usual sense, because the second order terms in N from the other terms are very small. This fact can be proved by a straightforward analytical calculation, if a typical liquid-drop mass formula is assumed as an example.

Let us consider the average neutron number $\bar{N}_i(Z)$ of each element Z . By identifying the average value with the maximum of the N distribution [Eq. (3)], we get

$$(\partial/\partial N)[G_{\text{nuc}}(N, Z) - \mu_{ni}N - \mu_{pi}Z]_{N=\bar{N}_i(Z)} = 0. \quad (5)$$

A straightforward calculation, using the specific form of G_{nuc} of Eq. (4), results in

$$C(Z)\{1 - 4[Z/\bar{A}_i(Z)]^2\} = \mu_{ni} - a(Z), \quad (6)$$

with $\bar{A}_i(Z) = Z + \bar{N}_i(Z)$. The equations for the two reaction systems, $i=1$ and $i=2$, subtracted side by side then yield

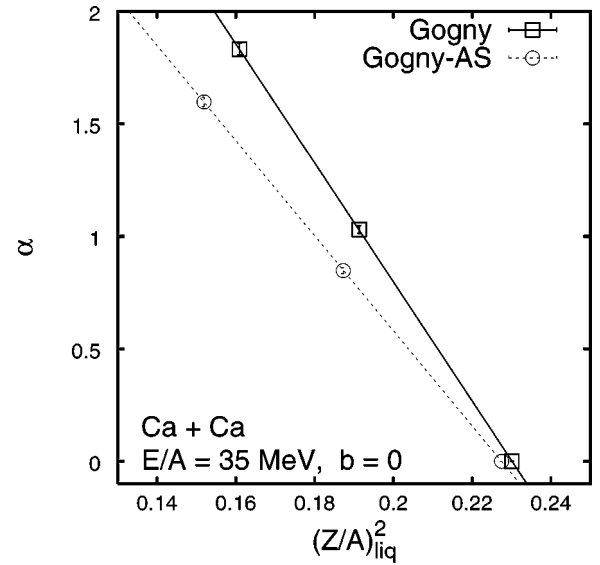


FIG. 6. Relation between $(Z/A)_{\text{liq}}^2$ and α for the three systems $^{60}\text{Ca}+^{60}\text{Ca}$, $^{48}\text{Ca}+^{48}\text{Ca}$, and $^{60}\text{Ca}+^{40}\text{Ca}$ (from left to right). Open squares and filled circles show the results of AMD with the Gogny force and Gogny-AS force, respectively. Both $(Z/A)_{\text{liq}}^2$ and α are calculated for fragments recognized at $t=300$ fm/c. The straight lines are drawn so as to connect the points for $^{40}\text{Ca}+^{40}\text{Ca}$ and $^{60}\text{Ca}+^{60}\text{Ca}$. The line slopes are -26.48 for the Gogny force and -21.16 for the Gogny-AS force, respectively.

$$\frac{\alpha}{[Z/A_1(Z)]^2 - [Z/A_2(Z)]^2} = 4C(Z)/T, \quad (7)$$

relating the isoscaling parameter α , the $(Z/A)^2$ of fragments, and the symmetry energy coefficient $C(Z)$ which is a function of (T, P) . Interestingly, this relation does not involve the terms in G_{nuc} other than the symmetry-energy term.

In the AMD simulations, the Z dependence of $Z/\bar{A}_i(Z)$ is rather weak for $Z \geq 5$, making it meaningful to consider the isospin asymmetry of the liquid part $(Z/A)_{\text{liq}}^2 \equiv (Z_{\text{liq}}/A_{\text{liq}})^2$ that has been averaged over all the fragments with $A > 4$. For this quantity we expect the relation

$$\frac{\alpha}{(Z/A)_{\text{liq},1}^2 - (Z/A)_{\text{liq},2}^2} = 4C/T. \quad (8)$$

In the end, we find that the linear relation observed in Fig. 6 is consistent with the equilibrium relation (8), suggesting applicability of statistical laws to the isospin composition of fragments even in dynamical models.

A relation similar to Eq. (7) has been derived in other statistical models [4,14] by assuming a single emitting source nucleus,

$$\alpha = 4C_{\text{sym}}[(Z_1/A_1)^2 - (Z_2/A_2)^2]/T, \quad (9)$$

where C_{sym} is the symmetry energy, T is the source temperature, and (Z_i/A_i) is the asymmetry of an equilibrated emitting source. Equation (9) cannot be used, however, to examine the AMD results because in the AMD simulations of collisions there is no easily identifiable single

emitting source nucleus. All fragments are emitted on about equal footing. Instead, Eqs. (7) and (8) provide the needed link to characterize the thermal properties of a dynamically evolved collision.

From the slope of the linear relation in Fig. 6, we obtain $4C/T=26.5\pm 0.4$ for Gogny force and $4C/T=21.2\pm 0.3$ for Gogny-AS force, where the shown uncertainties are due to statistics. From these values, we can extract the density and temperature pertinent to the fragmentation. The comparison of the ratio of the slopes $C(\text{Gogny})/C(\text{Gogny-AS})=1.25\pm 0.03$ and the density-dependent symmetry energies in Fig. 1 suggests that fragments are formed when $\rho\sim 0.08\text{ fm}^{-3}$, which is consistent with the idea that fragmentation occurs at a reduced density. Furthermore, by using the absolute value of C at this density in Fig. 1, we get the temperature $T\sim 3.4\text{ MeV}$, which is in a reasonable range.

When shifting the fragment recognition time t_{recog} within the range 150–300 fm/c, we find that *both* α and $(Z/A)_{\text{liq},1}^2 - (Z/A)_{\text{liq},2}^2$ are decreasing functions of t_{recog} , if the isoscaling fit is done for $Z\geq 3$ fragments only. Nonetheless, the extracted value of $4C/T$ is independent of t_{recog} within statistical uncertainties. However, if one tried to compare the results directly to the experimental data, the effect of the secondary

decay of excited fragments should be taken into account.

In conclusion, isospin fractionation and isoscaling are observed in the dynamical AMD simulation. A linear relation between the isoscaling parameter and the fragment isospin asymmetry $(Z/A)_{\text{liq}}^2$ is observed. Such linear relations suggest that the fragment isospin composition is subject to the statistical laws even in a dynamical picture of production. The relation also provides a means to relate the dynamical observable to thermal dynamical properties such as the temperature and density. The slope of the linear dependence is sensitive to the symmetry terms used in the nuclear equation of state and may yield information about the symmetry energy of the emitted fragments.

This work was supported by Japan Society for the Promotion of Science and the US National Science Foundation under the U.S.-Japan Cooperative Science Program (INT-0124186), by the High Energy Accelerator Research Organization (KEK) as the Supercomputer Project No. 83 (FY2002), and by grants from the US National Science Foundation, PHY-0245009, PHY-0070161, and PHY-01-10253. The work was also partially supported by RIKEN as a nuclear theory project.

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